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Aerodynamics of Jets Pertinent to VTOL Aircraft

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Nomenclature

A_j = area of the jet nozzle
A_p = surface area of the baffle plate
C = circumference of a jet in a cross flow
C_p = pressure coefficient defined by Eq. (4)
d_j = diameter of the jet nozzle
D_e = effective diameter of a multiple array of jets, the diameter of a circle with an area equal to the total area of the jet nozzles
D = diameter of the baffle plate (Fig. 2); diameter of a cylindrical body placed between the jets (Figs. 6, 8)
$\langle D \rangle$ = $(1/\pi)\int_0^{2\pi}\langle r\rangle d\theta$, where $\langle r\rangle$ is the distance from the center of the jet to the edge of the planform, and θ is the polar angle
E = entrainment parameter, the rate increase of mass flow in the jet per unit length along the jet
e = spacing between the jets in an array or between rows of jets
F = momentum thrust of the jet
g = gravitational acceleration
h = distance between nozzle exit plane and the wall
h' = distance between the wall and the virtual origin of the jet
h^* = height of the dust cloud above the point of impingement of the jet
K = const
K_0 = const [in the expression for the mass flow in a jet Eq. (1)]
L_i = lift component due to interference of the jet (Fig. 13)
ΔL = loss of lift at a height h above the ground
ΔL_∞ = ΔL as $h \to \infty$
M = $2\pi\rho\int_0^\infty u^2 r dr$, the integrated momentum flux of the jet
M_j = total momentum flux for one or more jets
M_u = integrated momentum flux in the upflow region between impinging jets
m = rate of mass flow in the jet
m_j = rate of mass flow in the jet at the nozzle exit plane
p = pressure
q = $\frac{1}{2}\rho u^2$
q_j = $\frac{1}{2}\rho u_j^2$, the dynamic head of the jet at the nozzle
q_m = $\frac{1}{2}\rho v_m^2$, the maximum dynamic head in the wall jet
q_0 = $\frac{1}{2}\rho u_0^2$
r = radial coordinate in a cylindrical system
r^* = radius at which the flow separates from the wall
T = temperature
u = axial velocity component
$\langle u \rangle$ = mean velocity of the jet in a cross flow
v = radial velocity component
x = x coordinate (axial coordinate in a cylindrical coordinate system)
y = y coordinate in a Cartesian system
z = z coordinate (axial coordinate in a cylindrical system)

Greek symbols

β = expansion coefficient of the gas
δ = width of a jet in a cross flow
ζ = natural curvilinear coordinate for the jet in a cross flow (Fig. 9)
η = natural coordinate for the jet in a cross flow (Fig. 9)
θ = local angle of inclination of a jet with respect to its initial direction normal to a cross flow
ξ = distance along the axis of a jet in a cross flow (Fig. 9)
ξ' = value of ξ measured from the virtual origin of the jet, chosen to make the curves in Fig. 10 coincide
ρ = mass density of the fluid in the jet

Subscripts

a = ambient conditions in environment
i = initial condition (Fig. 2)
j = evaluated at the nozzle exit plane
m = maximum value in the jet
o = freestream conditions
s = stagnation condition
w = evaluated at the wall
$\frac{1}{2}$ = property evaluated where the velocity is half the maximum value

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I. Introduction

It is a disturbing fact to those concerned with the aerodynamic design and testing of VTOL aircraft that the aerodynamic effects of propulsive jets providing lift forces during hover and transition flight are at once both significant and poorly understood. Under hover conditions, the entrainment of air by the jet causes a reduction of pressure on the lower surfaces of the airframe, resulting in a loss of lift. The freestream flow during transition flight enhances the entrainment effects and introduces other effects, typically resulting in a loss of aerodynamic lift and significant pitch-up moments. There are some conditions under which favorable lift effects may occur, but usually at the expense of increased drag. The proximity of the ground plane can enhance the aerodynamic effects both in favorable and unfavorable ways.

Although the development of VTOL aircraft in the United States and abroad has stimulated experimental and theoretical research on the problems, many questions remain unanswered. Some of the experimental and analytical results contributing to the understanding of the elementary flow phenomena are reviewed in the following. Investigations involving the simpler geometries are emphasized; more complex situations, such as the testing of protoytpe aircraft models, have been omitted. Preference is given to aerodynamic effects associated with circular, subsonic jets, although occasional citations are included for similar investigations of plane jets.

The most elementary flowfield appropriate to this discussion, that of an axisymmetric free jet, has been subject to extensive investigation.[1-8] Theories have been advanced leading to relations for the properties of the jet in the developed region, far from the source, predicting the dominant aspects of the flow with varying degrees of accuracy. But no completely acceptable theory exists for the entire flowfield of the jet, including the induced aerodynamic field outside the jet. The difficulty, of course, ultimately resides in the characterization of turbulence in free-shear flows. Semi-empirical theories have also been advanced for the initial mixing layer around the potential core of the jet[1,9] and for the transition region.[1,10] There, the descriptions are less complete and accurate, even accounting for the initial boundary layer on the wall of the nozzle. Notwithstanding some evidence to the contrary (see Sec. II), the theories of axisymmetric free jets are not entirely adequate for determining the induced aerodynamic field. The prospects for similar theories applied to the flow of a jet in a cross wind, then, would appear to be rather bleak, considering that there the shear layer is three-dimensional and contains strong streamwise vorticity as well.

Similar remarks might be made with regard to the axisymmetric jet in a quiescent atmosphere impinging normally on a ground plane. For that flow, there are fewer experimental data, a smaller number of different theories, and generally poorer agreement between the experimental data and the theories. Brief reviews of the wall jet flow, the turning region, oblique impingement, and particle entrainment are presented in Ref. 8.

The merits of the Glauert[11] theory of the turbulent wall jet appear to lie principally in that it predicts quite well the shape of the velocity profile, except near the outer edge of the jet.[12-15] The predicted variation of the flow properties along the jet, the wall shear stress, and other factors are not in complete agreement with observations.[12-19] There is also evidence suggesting that the turbulent transport model employed fails to represent the shear stress throughout the jet, notably near the peak velocity points in the jet.[14,16] The problem is complicated by the interaction between the jet-like flow and the flow near the wall, and remains a current topic in turbulent flow research. Related topics of interest, such as the influence of free convection normal to the wall jet for hot jets, have not yet been exhaustively investigated. A convenient annotated bibliography on wall jets has been compiled by Rajaratnam and Subramaya.[20]

The turning region involves a turbulent flow in two dimensions and has not been thoroughly studied as such. It has been consistently observed, however, that significant changes in the free jet begin to occur at a distance above the plate corresponding roughly to the diameter of the jet.[13,15,21] Approximate theories treating the flow as irrotational[22-30] or inviscid but rotational[31] have been most prominent. An extension of the Kolmogorov-Prandtl hypothesis of turbulence to the plane impinging jet flow has also been advanced.[32] It is possible to predict approximately the static pressure variation along the wall in the turning region with some of the previous theories, although the available experimental data are hardly consistent.[8] The wall boundary layer undergoes transition at some radial distance from the stagnation point. There is evidence that conventional methods for predicting transition are not entirely reliable for this problem because of the forcing influence of the freestream turbulence in the impinging jet[15,33] on the wall boundary layer.

The available relations for the properties of jets impinging at oblique incidence are primarily empirical correlations.[34,35] Few such investigations have been reported. Particle entrainment along the ground plane may also be approximately accounted for by empirical correlations.[36-38] The mechanics of the phenomena are not completely understood.

While in a strict sense, improvements in the theories of free and impinging jets imply improved understanding of the turbulence in free shear layers, it is not entirely conclusive that such detailed knowledge of the jet flows is required for computing the aerodynamic effects of the jets. Even if that happens to be the case there is merit in approximate theories which yield correct trends, but require empirical adjustment for quantitative purposes. Depending on the significance of the detailed structure of the jet on the aerodynamic effects, the latter is probably all that can reasonably be expected within the present status of turbulence theory, for problems associated with the aerodynamics of jets.

II. Induced Aerodynamic Field and Secondary Flows

The induced aerodynamic field outside the region of turbulent flow within the jet has been observed to possess negligible vorticity, at least by comparison with that in the jet. Theoretical relations for the induced flow have made use of the scalar velocity potential, subject to specification of the normal velocity component on the surface of the jet and on any geometric constraints present in the flowfield. The time-averaged values of the normal velocity component are related to the local rate of mass flow entrained by the jet. Relations for the statistical behavior of the time-varying surface of the turbulent jet have also been considered. All of those relations rely on knowledge of the turbulent jet flow that is difficult to obtain with any accuracy, either experimentally or theoretically, because of the sensitivity to the properties of the flow near the edge of the jet. An alternative method for such computations is to relate the induced flow to the time-averaged vorticity in the jet through the vector potential.[39] Computations involving the vector potential are more complex, and they apparently have not been applied to this problem.

The aerodynamic effects of a freejet in a quiescent atmosphere are considered in the following discussion. Those effects are normally rather small (although not necessarily insignificant), at least regarding the induced forces, unless the jet is confined in some way by the geometry of the airframe or the proximity of the ground plane. Aside from the induced forces, other important effects, such as recirculation of the exhaust gases to the engine inlet, are influenced by the induced aerodynamic field. Nonetheless, the principal motivation for examining such flows here is simply that there are more abundant data for the properties of jets exhausting into a quiescent environment.

A. Freejets

The available experimental evidence suggests that the presence or absence of a plane wall through which the jet exhausts has little effect on the properties of the jet, although the flow must strictly depend on the geometric constraints present. The pressure force on a disk with a diameter one or two orders of magnitude larger than that of the jet nozzle is only 1 or 2% of the momentum flux of the jet. The effect of the presence of the plate on the momentum flux in the jet, then, would be expected to be hardly measurable in a turbulent flow. That does not exclude the possibility that the entrainment of mass by the jet could be affected significantly, however, which would require only small variations in the mean velocity profile near the edge of the jet.

The general pattern of the induced aerodynamic field of a free jet may be inferred from the results of an exact solution of the Navier-Stokes equation for a jet-like flow obtained by Squire.[40] The solution represents the action of a concentrated force on the fluid at the origin of the coordinate system. Squire's solution, however, accounts for no mass flux or flux of vorticity (as from the separation point at the lip of a nozzle). Similar results were reported by Landau and Lifschitz.[41] They noted that the solution could be considered to be the first term in a series expansion, and extensions of the results along that line to account to the finite size of the jet origin have been studied.[42] For a jet exhausting through a plane baffle plate, the flow in the induced field far from the jet might be expected to be approximately radially inwards.[6]

Ricou and Spalding[43] summarized the available results for the mass flow entrained by a jet, based on measured velocity profile data and augmented by the existing analytical solutions. If the mass flow in the jet is expressed by

$$m = K_o (M/\rho_a)^{1/2} x \qquad (1)$$

they observed that the values in the literature for K_o ranged from 0.220 to 0.404. The highest value corresponds to the Schlichting[6] profile.

For the purpose of obtaining direct measurements of the entrained mass flow, Ricou and Spalding[43] conducted an experimental investigation wherein a jet was directed along the axis of a porous cylindrical chamber of variable length, exhausting to the atmosphere through an orifice at the end of the chamber. Gas was bled into the chamber around the jet through the porous wall to balance the pressure in the chamber with the ambient pressure. The mass flow entrained by the jet was then equated with the mass flow bled into the chamber. The experimental data showed Eq. (1) to be applicable for jet Reynolds numbers above 25,000. Their results for air jets into air yielded the relation

$$m/m_j = 0.32 x/d_j \qquad (2)$$

corresponding to a value of K_o of 0.282. The results of experiments with the density of the gas in the environment different from that of the gas in the jet were found to follow the relation

$$m/m_j = 0.32(\rho_a/\rho) x/d_j \qquad (3)$$

In contrast to the Ricou and Spalding results, Wygnanski[44] made use of the higher Schlichting value of K_o in computations of the pressure distribution on a plate normal to the jet nozzle, finding remarkably good agreement with his experimental data. Wygnanski took the increase in volume flow/unit length to be $0.201\, r_j u_j$ for $x/d_j < 6$ (the length of the potential core plus the transition region) and chose a value corresponding to $K_o = 0.404$ in Eq. (1) for $x/d_j > 6$. The increase in volume flow rate/unit length was taken to be equivalent to a linear distribution of sinks along the axis of the jet, thus neglecting the actual geometry of the jet. The potential

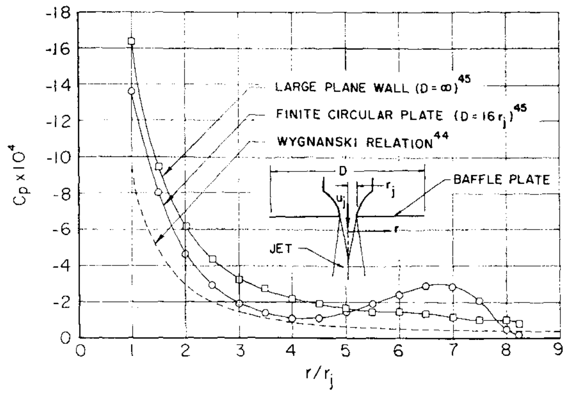


Fig. 1 Pressure coefficient induced by a jet on a circular baffle plate.

flow along the wall was computed from the sink distribution of the jet and an image opposite the jet. The results could be expressed in terms of the pressure coefficient on the plate as

$$C_p = \frac{p - p_a}{\frac{1}{2}\rho u_j^2} = -\frac{d_j^2}{16\pi^2 r^2}\left\{0.715 - \frac{0.514}{[1 + (r/6d_j)^2]^{1/2}}\right\}^2 \qquad (4)$$

At large values of r/d_j, Wygnanski suggested C_p should behave as $-0.00325 d_j^2/r^2$. Results were also obtained for a plane jet, again with excellent agreement between the theory and his experimental data.

It is apparent that some caution is required in extending data for the pressure distribution over a large baffle plate to situations where the dimensions of the plate are relatively small. Gentry and Margason,[45] for example, found secondary flow effects to be significant near the edge of a circular plate about eight times the diameter of the jet. Both the case where the plate was flush with a large plane wall and the case where the plate was exposed to the environment on both sides were studied. A comparison of their results for the two cases is shown in Fig. 1. A rise in the magnitude of the pressure coefficient near the outer edge of the plate occurred when the plate was exposed to flow induced around the edge of the plate from behind it. The plate flush with the wall experienced the larger total pressure force. Wygnanski's values according to Eq. (4) are also shown in Fig. 1. His results fall below those of Gentry and Margason by nearly 50%, for the case of the large plane wall.

Several factors have been found to influence the surface pressures induced on a baffle plate normal to the jet. For example, if the nozzle flow is distorted in some manner, if the initial level of turbulence in the jet flow is increased, or if a noncircular nozzle is employed, the pressure force on the baffle plate may be several times that for an aerodynamically clean flow from a circular nozzle.[45,46] Extension of the nozzle beneath the plate would be expected to reduce the pressure force on the plate by moving the effective aerodynamic sink away from the surface. There is evidence that extending the nozzle as much as three nozzle diameters beneath the plate has little effect, if any, on the force.[47] More recent data indicated a reduction in the force by as much as 50% as the nozzle was extended only one diameter beneath the baffle plate.[45]

Those investigations concerned with the details of the coupling between the induced aerodynamic field and the jet have led to statistical descriptions of the motion of the free surface of the turbulent jet and the effect of that motion on the region of irrotational fluid. Corrsin and Kistler[48] experimentally examined several types of turbulent flows bounded on at least one side by nonturbulent fluid, particularly the character of the intermittency near the free boundary of the jet. They found a Lagrangian diffusion analysis based on the statistical properties of the turbulence in the fully turbulent region of the jet predicted roughly the rate of increase in the wrinkle amplitude of the turbulent front. Phillips[49] and

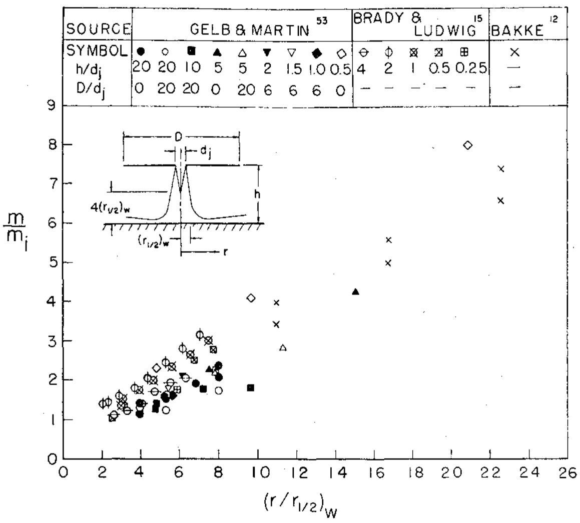


Fig. 2 Entrainment of mass in the wall jet.

later Stewart[50] developed analyses of the induced potential flow employing a stationary random fluctuation of the velocity component normal to the surface of the jet as a boundary condition. Stewart[50] carried through the analysis to yield relations for the velocity potential for a freejet with and without a plane baffle plate through which the jet exhausted. Recent interest in the entrainment characteristics of plane jets in connection with fluidics applications has led to the further investigation of the instabilities and vortex formation over the initial surfaces of the jet near the nozzle. The properties of successive rolled up vortices, insofar as entrainment of fluid from the environment is concerned, have been analyzed approximately, for example.[51]

The entrainment in free shear layers has been of interest in connection with separation bubbles and reattaching flows; there, the effects of curvature of the shear layer must be considered as it affects the entrainment. The inner region of a curved plane jet entrains significantly less fluid than the outer region.[52]

B. Impinging Jets

The irrotational flow induced by an impinging jet may be related to the rate of mass entrainment of the jet, as for the free jet. The same types of uncertainties in the rate of mass entrainment for the freejet exist for the impinging jet. There are even fewer experimental results and the aspect of the presence of the wall complicates the theory. The entrainment of mass by a wall jet may be estimated from both the theoretical relations and the experimental data. The Glauert[11] theory, for example, would predict the ratio of the mass flow rates in the wall jet at different radii to vary with the radius raised to about the 0.9 power. The rate of entrainment, then, expressed as the mass of fluid entrained/unit length along the wall, would be expected to decrease along the wall rather slowly. Most of the measured data, based on integration of the measured velocity profiles, show approximately a linear variation of the mass flow in the wall jet with increasing radius, as for the developed region of a free jet along its axis. However, these results are hardly conclusive.

The experimental data from Refs. 15 and 53, including points computed from the Bakke[12] data, have been plotted in Fig. 2. The radial coordinate has been normalized with respect to the radius of the freejet at the wall $(r_{1/2})_w$ estimated from the relation $(r_{1/2})_w = 0.0848\, h'$, where h' is the distance between the plate and the virtual origin of the jet. The mass flow rate m in the wall jet has been normalized with respect to the mass flow rate m_i in the impinging jet, evaluated at a distance 4 $(r_{1/2})_w$ above the wall for $h/d_j > 2.4$, according to Eq. (2). For smaller values of h/d_j, m_i was simply equated with m_j. That characterization provides a reference mass flow rate roughly corresponding to that entering the turning region and a reference length at the wall characteristic of the freejet. The plot shows the scatter that might be expected for results sensitive to the values in the outer region of the jet, and some variance introduced by uncertainties in m_i and $(r_{1/2})_w$. Some tendency for those points corresponding to cases where a baffle plate was present to fall below the rest of the data may be observed, as could be expected.

Harris et al.[54] noted that studies at Cranfield[55,56] have shown that a relation of the form

$$m/m_j = 0.68(r/d_j)(T_j/T_a)^{1/2} \tag{5}$$

holds for the rate of mass flow in a wall jet formed by an impinging axisymmetric jet. They apparently have not found a significant variation with h/d_j. Equation (5) falls within the scatter of the data in Fig. 2 if one takes $(r_{1/2})_w = d_j/2$, $m_i = m_j$, and $T_j = T_a$.

If the jet exhausts through a plane baffle plate normal to the axis of the jet, there are significant changes in the induced aerodynamic field as h/d_j is reduced because of the constraint of the induced flow. For large h/d_j, where the influence of the ground plane is not significant, the induced pressure force on the plate is typically only 1 or 2% of the momentum flux in the jet, as noted in Sec. II A. As h/d_j is reduced, the negative pressure force on the plate increases, eventually exceeding the jet momentum thrust at sufficiently small values of h/d_j.[59] The height above the ground below which the effect of the ground plane becomes significant depends primarily on the size and shape of the baffle plate. The available data indicate that, for a circular or square baffle plate, the height above the impingement plane below which significant changes in the pressure force occur is on the order of the characteristic size of the plate (1 or 1.5 diam for a circular plate).

A number of investigations have been concerned with evaluating the loss of effective jet thrust caused by the induced pressure forces on the baffle plate in the proximity of the ground. Several simple planforms of the baffle plate (circular, rectangular, and triangular), as well as scale models of VTOL airframes have been studied with both single and multiple circular and rectangular jets.[45,47,57,58] Wyatt[57] found that an expression of the form

$$(\Delta L - \Delta L_\infty)/F = 0.012\, [h/(\langle D\rangle - d_j)]^{-2.36} \tag{6}$$

correlated virtually all of his experimental data for circular, triangular, and rectangular planforms with a single jet centrally located in the planform. The experiments were conducted with circular baffle plates for which d_j/D ranged from 0.108 to 0.292, and with triangular and rectangular plates of equal areas corresponding to that of the circular plate with $d_j/D = 0.108$. The height of the baffle plate above the ground was varied from 0.15 to about 1 plate diameter. For the circular plates, Eq. (6) agreed with the data within ± 0.01. The measurements were made with several jet velocities ranging from 400 to 980 fps. While both the Wyatt[57] and the NACA[47] data corresponded to relatively small-scale experiments ($d_j = 4.4$ and 1.0 in.) with air at ambient temperature, Hall[59] found that the correlation given by Eq. (6) represented the data taken in a full-scale experiment with a J-85 turbojet engine exhausting through a square baffle plate.

The details of the flowfield have been examined by flow visualization and by means of static pressure measurements on the baffle plate, and some velocity measurements have been taken. Changes in the static pressure distribution on a circular disk as h/d_j is reduced are illustrated in Fig. 3a. The results in Fig. 3 were obtained by Yakovlevskiy and Sekundov[60] for a circular baffle plate 190 mm in diameter with a nozzle 20 mm in diameter. The nozzle exit velocity was 88 m/sec. Similar measurements for square plates were reported by Spreeman and Sherman,[47] for a full-scale experiment with a square plate by Hall,[59] and for a circular plate by Gelb and Martin.[53]

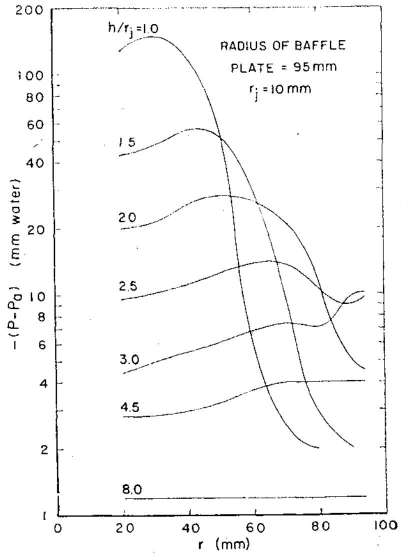
The general character of the flow pattern as observed by flow visualization[60] is shown in Fig. 3b for a circular disk. A stationary ring vortex exists near the baffle plate, its position and extent varying with the height of the plate above the ground. When the plate is sufficiently close to the ground, the wall jet attaches to the plate forming a closed secondary flow region with no entrainment of external air.

When the gases in an impinging jet are hot, as in the exhaust of a gas turbine engine, the flow along the wall may be observed to separate from the wall due to the buoyancy effect. Experiments with heated jets by Cox and Abbott[19] showed that the horizontal distance along the wall to the point of separation could be correlated by the expression

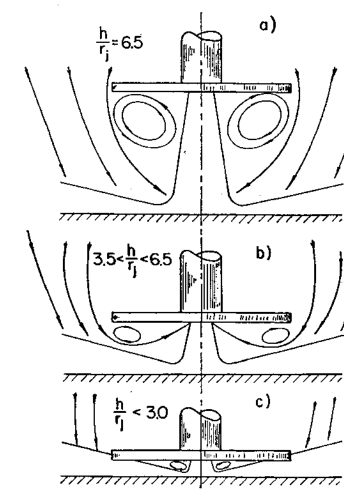
$$\frac{r^*}{d_j} = 0.62\left[\frac{u_j^2}{g\beta(T_j - T_a)d_j}\left(\frac{T_a}{T_j}\right)^{1/2}\right]^{1/2} \tag{7}$$

where β = the expansion coefficient of the gas, $1/T_a$. An approximate analysis was suggested for the phenomenon, relating an effective vertical pressure gradient to some fraction of the local maximum dynamic head in the jet at the point of separation.

Separation of the jet from the ground also occurs when there is an external flow or surface wind along the ground. Abbott[34] found that the point of separation could be associated with the point where the maximum velocity in the steady wall jet was about half the magnitude of the external flow velocity. A plot of $(q_o/q_m)^{1/2}$ vs $(q_o/q_j)^{1/2}$ including data from small-scale tests and several full-scale tests with taxiing aircraft showed $(q_o/q_m)^{1/2}$ to be 0.5 ± 0.1 for $(q_o/q_j)^{1/2}$ from 0 to 0.08, with no consistent variation about that line. Data were also obtained for the height of the exhaust cloud blown back over the point of impingement by the surface wind. The data for the height of the cloud over the point of impingement fell between $h^*/r^* = 0.5$ and 1.0 for the case where $h/d_j = 4$, including data for oblique impingement of the jet. The results were all obtained by observing a cloud of dust entrained in the jet along the wall.



a) Variation of static pressure on the plate as a function of h/d_j



b) Observed flow patterns

Fig. 3 Impinging jet beneath a baffle plate (Ref. 60).

III. Multiple Jets and Nonuniform Jets

By comparison with the single axisymmetric jet and the plane jet, relatively little is known about the interactions between multiple jets and the turbulent exchange phenomena in three-dimensional jets from nozzles of complex geometry. Because of their practical applications, however, there have been investigations to evaluate some of the relevant properties of multiple jet arrays, nonuniform jets, and jets from nozzles of complex geometry.

A. Freejets

When two or more circular jets exhaust parallel to each other, they eventually merge, forming a single, three-dimensional jet. The merged, larger jet eventually tends to become axisymmetric. Experiments with rectangular jets,[61] for example, show that the cross section of the jet tends initially to become roughly elliptical, deforms through one or more cycles whereby the positions of the major and minor axes of the ellipse are reversed, and eventually becomes axisymmetric. A similar behavior might be inferred for three-dimensional jets of lower symmetry. For multiple jets flowing parallel to each other, elementary considerations suggest that even before the turbulent regions of the jets merge, there will be at least a weak interaction between the jets. Each jet experiences the effects of the flowfield induced by the others. There would be expected to be an attraction between the jets causing their axes to curve slightly and the symmetry of each jet to become somewhat distorted.

The detailed computation of general, three-dimensional, turbulent free-shear flows at subsonic speeds is encumbered by lack of knowledge of the turbulent transport properties for such flows. Three-dimensional flows with simple symmetry have been examined by Sforza and others,[61,62] and for those problems, a mixing model based on an extension of Prandtl's model appears to yield useful results.

Another approach for treating multiple parallel jets and similar flows, suggested by Alexander et al.[4] involves simply superposing the axisymmetric flows in a linear manner. They considered the equation of motion for a jet originating from a point source in terms of the kinematic momentum flux, following the analysis of Reichardt.[63] The Reichardt hypothesis relating the correlation $\overline{uv}$ to the derivative of the kinematic momentum flux $\overline{u^2}$ renders the equation of motion in the axial direction linear. They proceed to apply the solution

$$\overline{u^2} = [K/b^2(x)]\exp\{-[r/b(x)]^2\} \tag{8}$$

where $b(x) = 0.075x$ and K = const(related to the kinematic momentum flux of the jet), to complex problems, assuming strong interactions do not occur, by superposing the solutions for single jets. The results were found to agree quite well for such problems as the initial mixing layer of an axisymmetric jet, a pair of parallel circular jets, and a single circular jet exhausting parallel to a plane wall. The superposition of solutions does not yield useful results for strong interactions, however, as would occur in the impingement of two nonparallel jets.

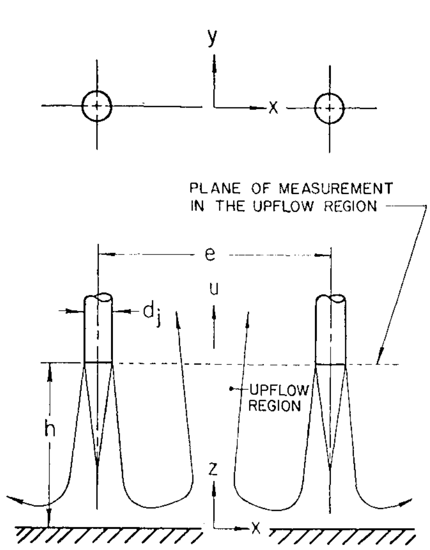


Fig. 4 Flowfield formed by two jets impinging on a wall.

When several circular jets exhaust parallel to each other, the decay of the peak velocity is more rapid and there is more entrainment of fluid from the environment than for a single circular jet of a cross-sectional area equal to the total area of the multiple jets. The induced pressure force on a baffle plate through which the jets exhaust, accordingly, is larger than that for an equivalent single jet. Gentry and Margason[45] found the pressure forces for arrays of 4 and 8 jets to be as much as four times the value for a single jet with equivalent total flow conditions. They were able to correlate their data according to the relation

$$\Delta L/F = 0.009\{(A_p/A_j)\partial[q/(p_{sj} - p_a)]/\partial(x/D_e)|_{max}/(x/D_e)_i\}^{1/2} \quad (9)$$

where $\partial[q/(p_{sj}-p_a)]/\partial(x/D_e)$ = the maximum slope of the decay curve for the normalized peak dynamic head in the jet and $(x/D_e)_i$ = the normalized distance from the nozzle corresponding to the position of the maximum slope of the decay curve. Their data included runs with a number of different planform shapes for the baffle plate. Results for the decay of the peak dynamic head in arrays of 4 and 8 circular jets and in an array of 4 rectangular jets were included in Ref. 45. Similar decay data have been obtained for some seventeeen exhaust nozzles designed for rapid mixing, including single rectangular nozzles, segmented circular nozzles, and linear arrays of four rectangular nozzles of varying aspect ratio and spacing.[64,65] Full-scale runs[66] with a J-85 engine, adapted for multiple exhaust ports, were found to agree with the small-scale, cold-flow data obtained by Gentry and Margason, agreeing with Eq. (9) within ±20%.

The flow in the downwash of a propeller or a rotor may be considered as a nonuniform (approximately annular) jet,

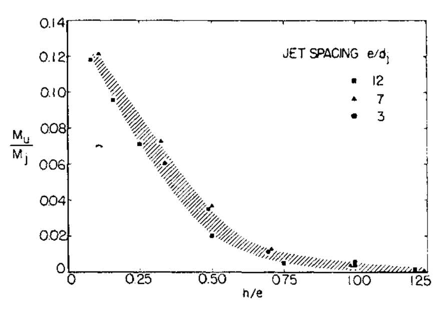


Fig. 5 Fraction of the initial momentum flux in the jets occurring in the upflow region for two impinging jets (Ref. 70).

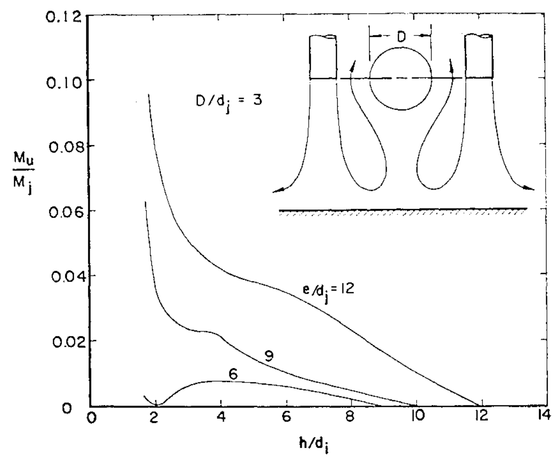


Fig. 6 Influence of a cylindrical body on the momentum flux in the upflow region of two impinging jets (Ref. 70).

although the flow differs significantly from that discharging from a nozzle. There is a rotational component of the motion, and the regions of turbulence are those formed in the wakes of the blades, occupying more or less spiral sheets in a predominantly inviscid flowfield. The theory of the flow in the downwash has been developed primarily on the basis of inviscid flow models involving rather strong assumptions regarding the flow geometry.[67-69]

B. Impinging Jets

When two or more parallel jets impinge normally on a plane surface the resulting flowfield exhibits the normal behavior along the wall and, in addition, the wall jets from the different sources meet forming vertical upflow regions, as shown schematically in Fig. 4. Jets having sufficiently nonuniform velocity profiles, such as the downwash from a single rotor, also exhibit re-entrant upflows, in that case, a flow upward along the axis of the rotor. The character of the upflow region has been found to be quite sensitive to small imbalances between the jets and angles of inclination, and appears to be unstable under some conditions. The upflow region or fountain is a three-dimensional, turbulent free-shear flow and, accordingly, has received little theoretical attention. There have been a number of qualitative experimental studies of the phenomena, mainly related to specific aircraft problems. A few quantitative results are available, however, mainly those of Hertel and Harmsen.[70] They are discussed briefly below.

A number of configurations of parallel circular jets impinging normally on a flat plate were investigated by Hertel and Harmsen,[70] including 2 jets, 4 jets in a square array, 5 jets (a central jet in a square array of 4 jets), and arrays of 12 jets arranged in two parallel rows of 6 jets each. The spacing between the jets and the height above the ground plane were varied in the experiments. Results were obtained where no blockage of the upflow was present, and also where a cylindrical body representing an aircraft fuselage was placed between the jets, tending to block the upflow. A flow visualization scheme involving a slit of light and small tracer particles introduced into the flow was employed to obtain information regarding the induced flowfield and the upflow region. Pitot tube measurements were also made in the upflow region to determine the velocity distribution, primarily in the plane of the exhaust nozzles.

For two parallel, impinging jets, the distribution of velocity in the upflow, measured in the plane of the exhaust nozzles (refer to Fig. 4), was employed to yield the total integrated momentum flux in the upflow. A rough correlation of the fraction of the initial jet momentum flux in the upflow region could be obtained as a function of the ratio h/e, shown in

Fig. 5. The total momentum flux reaches about 12% of the initial jet momentum flux at h/e equal to about 0.1.

In order to evaluate the influence of a fuselage-like object on the upflow, a cylindrical body was placed parallel to the ground, between the jets, and with its axis in the plane of the exhaust nozzles. Figure 6 shows results for the total momentum flux observed in the upflow region in the plane of the exhaust nozzles. The total momentum flux was found to be generally reduced for all of the cylinder diameters employed, except at the larger h/d_j values. A cylinder having a diameter equal to about half the spacing between the jets was found to reduce the momentum flux in the upflow region to rather small values for all $h/d_j > 2$, however.

For 4 jets in a square array, the upflow region has roughly the form of two intersecting plane jets, the cross section having the form of a cross. A weak vortex core at the center of the configuration resulted from slight imbalances of the jets. The results for the total momentum flux in the upflow region, measured again in the plane of the nozzles, are presented in Fig. 7. It may be noted that for comparable nozzle spacing e/d_j, the fractional momentum flux in the upflow region persists at larger values for much larger heights above the plane of impingement as compared with the results for the case of 2 jets (Fig. 5). When a cylindrical body was positioned in the plane of the jets, the results of the measurements in the plane of the nozzles for the total momentum flux in the upflow were found to correlate roughly as shown in Fig. 8, as a function of h/e.

Similar results were found for several configurations of 12 jets arranged in two parallel rows. There, the upflow approaches a two-dimensional jet configuration. The total momentum flux in the upflow region is on the order of that shown for 4 jets, but varies with h/d_j in a manner sensitive to both the spacing between the rows of jets and the spacing between jets in each row. The correlation of the momentum flux in the upflow where a cylindrical body is present between the rows of jets is nearly identical with that shown in Fig 8.

Instabilities were observed in the upflow region due to the induced aerodynamic fields of the jets; slight imbalances would apparently cause the upflow region to be deflected toward one or another of the jets. Similar results were found by Adarkar and Hall[71] using various methods of flow visualization. They also found that slight imbalances of the jets, either due to differences in the nozzle pressure ratios, or due to slight inclinations of the jets, caused a deflection of the upflow toward one of the jets. Measurements of inlet temperature fields in the vicinity of the upflow also showed a fluctuating character.

When multiple jets exhaust through a baffle plate parallel to the ground, there is a complex interaction between the induced flow and the upflow in ground proximity. The resulting pressure force on the baffle plate may be either positive (upward for a downward-directed jet) or negative, depending on the configuration of the jets, the geometry of the baffle plate, and the height above the ground.

The results of some experiments indicate that arrays of jets approaching an annular jet configuration, such as a circular array of jets, tend to yield a favorable ground effect, the lifting force increasing as the distance from the ground decreases.[72] A propeller or a rotor tends to produce an annular flow, and it is well known that the lift of a rotor is increased near the ground.[69,73] At sufficient distances from the ground plane, arrays of discrete jets and annular jets decay to the form of a single axisymmetric jet. For that condition, the favorable ground effect would not be expected to occur. Experiments with a nonuniform, axisymmetric jet simulating the downwash of a propeller (except for the rotational motion), with higher velocities near the perimeter of the nozzle, showed a recirculation region near the point of impingement on the wall.[74] There was a closed vortex ring standing above the stagnation point at a nozzle height h/d_j equal to about 2, the extent of the recirculation zone decreasing with both higher and lower values of h/d_j. Measurements in the wall-jet region of that flow were found to be similar to those found for a uniform jet.

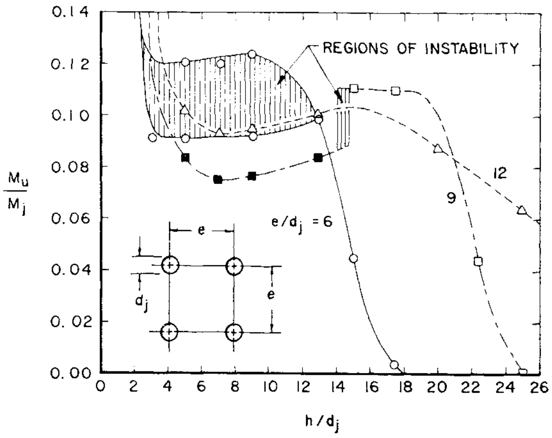


Fig. 7 Momentum flux in the upflow region for a square array of four jets (Ref. 70).

Experiments with a number of configurations of multiple jets have been reported,[75] with primary emphasis on obtaining the aerodynamic forces and moments on a plate through which the jet exhausts, in ground proximity. It would be difficult to generalize the results, however, because of the significant role of the geometry. The experimental data include results for the situation where a flow parallel to the ground was imposed on the system. While the results in the absence of the cross flow showed both increases and decreases in the lifting force, the influence of the cross flow tended uniformly to reduce the net lifting force.

IV. The Turbulent Jet in a Cross Flow*

Referring to Fig. 9, the turbulent jet in a cross flow is deflected downstream, the cross section of the jet deforming into a kidney-shaped section. Because of the shearing action of the external flow on the jet, secondary flows in the form of streamwise vortices are produced. The streamwise vortices tend to decay relatively slowly along the jet, dominating the flow in the downstream region. Although there is not much

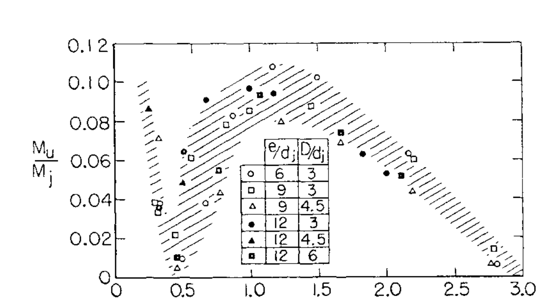


Fig. 8 Influence of a cylindrical body on the momentum flux in the upflow region of an array of four impinging jets (Ref. 70).

* This review includes the published literature through 1968. Additional papers relevant to the subject, especially Sec. IV, may be found among the papers given at the AIAA/AHS/VTOL Research, Design, and Operating Meeting, Feb. 1969, reporting on the two Themis Projects. Proceedings of a NASA Symposium on the subject "Analysis of a Jet in a Subsonic Crosswind," held in Sept. 1969, were published as NASA SP-218. The dominant themes of the more recent work are included in the review. Some new computations and experimental data may be found in those sources, however.

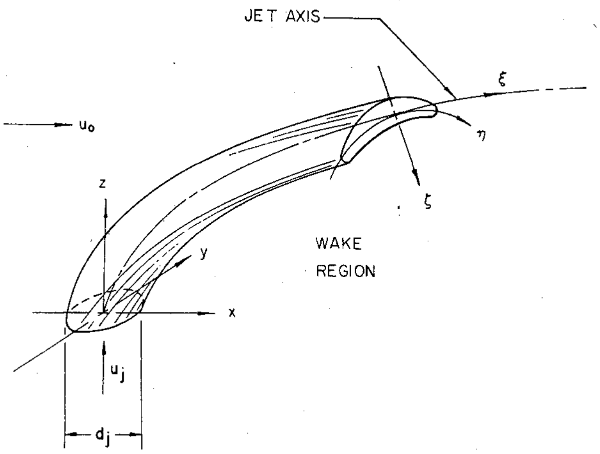


Fig. 9 Turbulent jet in a cross flow.

information available concerning the region behind the jet, there is likely to be a turbulent wake in that region, at least at the higher freestream-to-jet velocity ratios.

A. Experimental Data

Keffer and Baines[76] reported experimental studies of a subsonic air jet exhausting normally from a flat plate into a low-speed cross flow. They explored the mean velocity field and the intensity of turbulence in the jet by means of a constant-current hot-wire anemometer. Their measurements indicated that the distortion of the initially circular jet into the characteristic kidney-shaped cross section occurred in a distance along the jet roughly equal to the length of the potential core. For ratios of the jet velocity to the freestream velocity higher than about 4, the potential core remained approximately conical, but was roughly half as long as that for a freejet in a quiescent atmosphere. The jet appeared to retain the shape reached at the end of the potential core at positions farther downstream. They discussed the observation of relatively strong streamwise vortices developing behind the jet and parallel to the axis of the jet; the detailed structure of the velocity field in that region was not reported. Their data indicated a region of large-scale turbulence behind the jet, corresponding to the wake mentioned previously.

The measured velocity profiles in the jet, plotted as $(u\text{-}u_o)/(u_m\text{-}u_o)$ vs $\eta/\eta_{1/2}$, exhibited approximate similarity in the sense that no large or consistent variation of the profiles with different ratios of the jet-to-freestream velocity, or with different positions along the developed portion of the jet, were found. The peak velocity in the jet, as shown in Fig. 10, decays much more rapidly than that for a simple freejet, reflecting more rapid mixing with the environment. The intensity of the axial component of the turbulence was found to be 2–3 times that for a simple freejet ($u_0 = 0$) for the first 6–8 nozzle diameters downstream from the end of the potential core, eventually becoming lower than that for a simple freejet farther downstream.

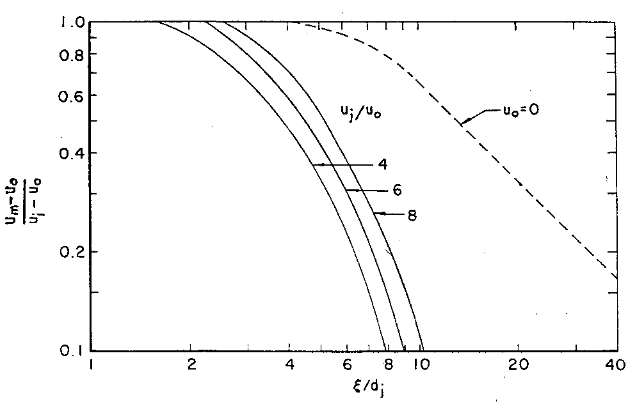


Fig. 10 Decay of the maximum velocity along the axis of a jet in a cross flow.

Measurements employing a pitot tube have been made to determine the stagnation pressure in the jet and in the field around the jet. Results of such surveys were reported by Jordinson[77] and data from Shandorov[78] were discussed by Abramovich.[1] Jordinson[77] mapped the stagnation pressure in a series of planes normal to the direction of the freestream and also in two planes normal to the axis of the jet. The contours of constant stagnation pressure are kidney shaped, roughly as shown for the section of the jet in Fig. 9. Shandorov's[78] data for the constant velocity contours, as presented by Abramovich,[1] were generally similar in shape to Jordinson's isobars, except as to some of the detailed structure.

Perhaps the most prominent observable property of the jet is the trajectory of its axis, representing the locus of points of maximum velocity in the jet. Measured values for various parametric conditions are shown in Fig. 11. The range of the data available is rather limited, extending less than 17 nozzle diameters downstream. The correlation suggested by Hurn and Akers[79]

$$x/d_j = 2.3\,(zu_o/d_ju_j)^3 \qquad (10)$$

appears to represent the data rather well for the range of downstream distances considered. The data show appreciable scatter, however, notably for small distances from the origin of the jet ($x/d_j < 5$). The scatter may be due partly to the sensitivity of the flow in that region to the initial jet conditions (nozzle, plenum) and to the influence of the wall boundary layer. It might be noted that Shandorov's experiments were apparently made with flow from a port in a thin-walled duct rather than from a nozzle. Because of the scatter in the data, use of Eq. (10) to obtain the local radius of curvature of the jet for approximate theories certainly does not seem warranted. Margason[80] obtained information pertaining to the deflection of a jet at a number of angles of inclination with the freestream by photographing the jet, made visible by introducing atomized water droplets in the jet flow upstream from the nozzle.

Data representing the lateral spreading of the jet according to Keffer and Baines[76] and Vakhlamov[81] are shown in Fig. 12. The Keffer and Baines[76] data represent the half width of the jet (along the curvilinear coordinate η). The growth appeared to be somewhat more rapid than that for a corresponding free jet,[8] although the growth was essentially linear with distance along the axis of the jet. Vakhlamov's[81] data correspond to the outer boundary of the jet, rather than the half width. Whereas the range of the data was rather limited, the growth of the boundary given by Vakhlamov's data appear to approach a linear variation with distance along the axis of the jet (dashed line in Fig. 12). The aspect ratio of the cross section of the jet (the ratio of the width of the jet to the thickness of the jet), as based on Vakhlamov's data, was roughly 5:1. Values from 4:1 to 5:1 have been employed for various analytical purposes in the literature (applying to the region of the jet beyond the potential core).

The aerodynamic influence of a jet exhausting normally from a plane surface, such as that of a wing, results in a general reduction of pressure on the surface, the negative interference increasing with increasing freestream velocity. There have been a number of investigations of different aircraft configurations and some flat-plate planforms.[82–84] The results of those investigations are useful for design purposes but are of limited value in clarifying the mechanics of the problem because the interest has been primarily to determine the induced lift forces and moments, as opposed to more detailed aspects of the flow. Representative results for a delta wing model are presented in Fig. 13 from Ref. 82, showing the

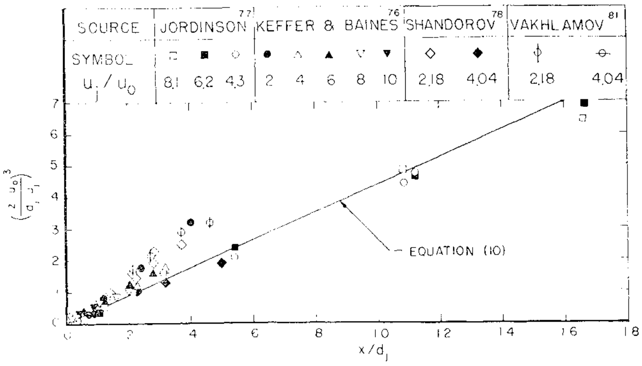


Fig. 11 Trajectory of a jet in a cross flow.

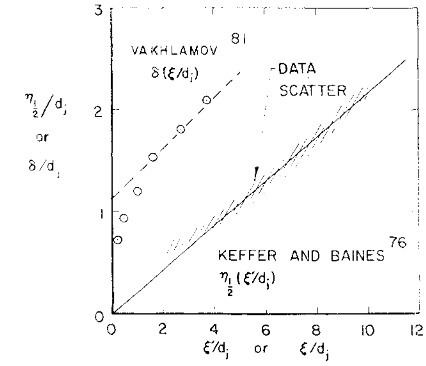


Fig. 12 Growth of a jet in a cross flow.

induced lift force as a function of u_o/u_j. The point made in all of the investigations is that the aerodynamic interference of a jet is extremely important, and that it is primarily sensitive to the geometric position of the jet, the jet-to-surface area ratio, and the jet-to-freestream velocity ratio. The induced negative lift force increases with both decreasing jet-to-surface area ratio and decreasing jet-to-freestream velocity ratio. There are some conditions under which favorable lift effects may occur (usually at the expense of increased drag). Significant pitch-up moments normally are experienced, which relate to the control and stability problems in V/STOL aircraft design. The induced aerodynamic field can also affect the flow around the tail control surfaces, again bearing on the control problems of the aircraft.

Several investigations[53,82,83,85,86] have been concerned with measuring the surface pressures around a single circular jet exhausting normally from a plane wall into a cross flow. There is a region of increased pressure upstream from the jet and a more extensive region of reduced pressure downstream. For lower ratios u_o/u_j, the region of reduced pressure tends to expand forward, ultimately causing the region of increased pressure to vanish altogether, as would be expected in the approach to the case where the jet exhausts into a quiescent atmosphere. Comparative measurements have been made for the flow around a solid cylinder showing roughly similar contours for the pressure coefficient, but there were significant quantitative differences and differences in the shapes of the contours, especially behind the jet.

B. Analyses

As might be expected, the theoretical treatment of the turbulent jet in a cross flow is limited to rather rough approximations because the turbulent transport properties of the three-dimensional free-shear flow are not known, and because of the geometric complexity of the problem. There are several approximate theoretical models for estimating the trajectory of the axis of the jet, and a number of analytical models have been suggested for computing the aerodynamic interaction of the jet with the external flow in the sense of determining the induced potential flowfield. Most of the analytical models employ line singularities (sinks, doublets, vortex lines) or approximations thereto for characterizing the jet in the aerodynamic field. The local strengths of the singularities are, in turn, related to postulated models for the flow. While all of the treatments are of a semiempirical character, some have met with reasonable success in accounting for the induced aerodynamic effects for particular applications. None accounts for the turbulent transport phenomena directly, even in an integrated fashion. Where the dynamical interaction between the jet and the external aerodynamic field is concerened, the theories substantially rely on matching the experimental jet trajectories through disposable constants in the models (drag coefficients, entrainment factors, and the like). The merits of the theories in treating broad classes of flow situations remain to be established (e.g., oblique incidence, strong interactions with the nonuniform field due to high-lift wing configurations, etc.).

The momentum balance associated with the jet flow has been treated in several ways. Abramovich[1] described a model considering the momentum conservation along the jet and normal to it. The streamwise momentum flux was assumed to remain constant, equal to that at the origin of the jet. The normal force acting on the jet was assumed to be expressed in terms of a drag coefficient ($C_D = 3$), the component of the freestream ve ocity normal to the jet, and an effective width of the jet given by the empirical relation.

$$\delta = 2.25d_j + 0.22\xi \tag{11}$$

A similar analysis by Vakhlamov[81] considered the momentum balance in the freestream direction and normal to the freestream flow. The jet was presumed to have a stagnant wake of dimensions corresponding to the downstream shadow of the jet, and the momentum flux normal to the undisturbed freestream flow was taken to remain constant. Both of the above models yielded results agreeing with the Shandorov[78] data for the jet trajectory within 10–20%.

Most of the more recent analytical models consider the momentum balance along the jet and normal to it. Wooler et al.[87] account for the normal force by employing a drag coefficient ($C_D = 1.8$) and also account for the effect on the normal momentum balance due to the entrainment of momentum flux by the jet. They further consider the change of streamwise momentum flux in the jet by equating the local rate of increase to the product of the rate of mass entrainment and the streamwise component of the undisturbed freestream flow. Williams and Wood[83] relate the strength of a streamwise vortex doublet to the interaction between the tangential vorticity in the jet (considered to be equivalent to a vortex sheet) and the freestream flow, given the trajectory of the jet. The change of momentum flux along the jet was neglected on their model.

Another approach[88–90] has been to relate the aerodynamic interaction to an effective blockage of the freestream by the

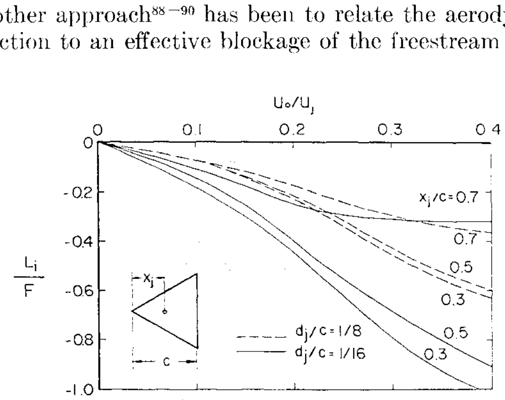


Fig. 13 Reduction of lift of a delta wing due to the aerodynamic interference of a jet exhausting normally from the wing (Ref. 82).

jet, considered as a solid body, but without further consideration of the mechanics of the flow. The arbitrary postulation of a rate of mass entrainment corresponding to that for a simple free jet has also been considered in those models.[91]

Probably the earliest attempts to describe the interaction between a jet and a cross flow in some detail were those made in Göttingen,[92] where the problem was treated from an inviscid point of view as the distortion of a vortex sheet separating the jet from the freestream. The problem was treated as a transient one, replacing distance along the jet by time, similar in some respects to the analysis of the rolling up of the trailing vortex sheet from a finite wing.[93] The distortion of the jet into the familiar kidney-shaped section is predicted, although comparisons with experimental data do not appear to be available. Some investigations along these lines are currently in progress, although no published results have come to our attention.[94] The theory is primarily applicab'e to the region of the jet near its source, where the flow is substantially normal to the freestream, but even there the curvature of the jet should probably be considered in the mechanics of the problem.

The model introduced by Wooler et al.[87] was based on a postulated relation for the rate of mass entrainment by the jet, containing three disposable constants. The constants were chosen to provide agreement with the Ricou-Spalding[43] data for the free jet in a quiescent atmosphere and to obtain agreement of the computed trajectories of the jet with the data of Keffer and Baines[76] and Jordinson.[77] The entrainment of mass/unit length, according to the model, was given as

$$E = 0.35\rho u_o \, \delta \cos\theta + 0.08\rho C(\langle u \rangle - u_o \sin\theta)/[1 + 30(u_o \cos\theta/\langle u \rangle)] \quad (12)$$

As noted previously, the streamwise and normal momentum relations were treated to an approximation in the model. The flow in the jet was characterized in terms of an average velocity and the cross section of the fully-developed region of the jet was assumed to be elliptical with an aspect ratio of 1:4. The deformation of the cross section of the jet in the initial region, from a circular section to the elliptical shape, was approximated in the solution. For the purpose of computing the aerodynamic field of the jet, the jet was represented by a distribution of doublets along the axis to represent the blockage effect, and a line of sinks normal to the plane of the jet axis in a surface containing the axis to account for the entrainment of the jet. The strengths of the singularities were computed locally along the path of the jet in the course of satisfying the continuity and momentum relations along the jet and normal to the jet. Computations for a finite wing, based on a combination of the model for the jet and a current lifting surface theory, appeared to yield remarkably good agreement with experimental data, considering the difficulty of the problem.

With regard to the determination of the flowfield of a finite wing with a jet or jets exhausting from it normal to the freestream, Williams and Wood[83] note that the induced downwash may be too large to be accurately treated by existing surface theory. Surface pressure distributions measured on a finite wing revealed significant differences from the results obtained with a jet exhausting from a large plane wall, notably a more extensive region of increased pressure upstream from the jet. The differences observed between the surface pressures with and without the jet did not appear to be subject to interpretation by any linear theory.

V. Conclusion

The aerodynamic effects of jets appear to be predictable only with a high degree of uncertainty. Even for those situations where the flow in the jet proper may be known empirically or analytically, computations of the aerodynamic field based on entrainment by the jet suffer severely from unavoidable uncertainties in the data. The more complex flows pose additional problems both empirically and analytically.

Complete analytical characterization of the flowfield, including that within the jet is hardly a realistic objective. Rather, theories for aerodynamic purposes should probably accept global conservation and make use of local approximations as necessary and acceptable. Alternative theoretical methods could be explored, such as the use of the combined vector and scaler potentials, to reduce the sensitivity of the computations to the inaccurate regions of the jet data.

Reliable and complete experimental data for a limited range of flow conditions would be most helpful in formulating theoretical models for the elementary problems. The degree of coupling between the aerodynamic field of the jet and that of finite airfoils with and without circulation could be explored further experimentally. Regimes of strong and weak interaction conditions could be assessed, for example.

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